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SPECIAL FEATURES OF THE SHOCK-WAVE STRUCTURE IN MIXTURES OF GASES WITH DISPARATE MOLECULAR MASSES

G. A. Ruyev,[1] A. V. Fedorov,[1] and V. M. Fomin[2]

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Asymptotic solutions are constructed for the problem of the shock-wave structure in mixtures of gases with disparate molecular masses. The effect of emergence of a plateau on the density profile of the light component and nonmonotonicity of the temperature profile of the heavy component is described. Based on a comparison with calculations of the full model, the range of applicability of asymptotic solutions is determined.

Mixtures of gases with disparate molecular masses posseses substantial velocity and temperature nonequilibrium because of the large inertia of heavy molecules. This is most clearly manifested in a shock wave, where the difference in velocities and temperatures of the components is of the order of their magnitudes. In this case, the flow should be described by equations of multivelocity, multitemperature gas dynamics, where each component has its own velocity and temperature.

The problem of the shock-wave structure in binary and ternary mixtures of gases was solved numerically on the basis of equations of multivelocity, multitemperature gas dynamics [1, 2]. A comparison with experimental data and calculations by kinetic equations shows that the equations considered yield a satisfactory description of the shock-wave structure in a wide range of molar concentrations of the heavy component for Mach numbers M = 1–4. Some features of the shock-wave structure considered in [1, 2] have not been adequately explained.

The objective of the present work is to obtain asymptotic solutions of the problem of the shock-wave structure in gas mixtures with low concentrations of heavy components and to explain effects observed in the shock wave on the basis of these solutions.

1. Physicomathematical Formulation of the Problem of the Shock-Wave Structure. The system of equations that describes a one-dimensional flow of a gas mixture has the form [1]

$$\frac{\partial\rho_i}{\partial t}+\frac{\partial\rho_i u_i}{\partial x}=0,\quad \rho_i\frac{\partial u_i}{\partial t}+\rho_i u_i\frac{\partial u_i}{\partial x}+\frac{\partial p_i}{\partial x}=\sum_{j(i\neq j)}K_{ij}(u_j-u_i)+\frac{4}{3}\frac{\partial}{\partial x}\Big(\mu_i\frac{\partial u_i}{\partial x}\Big),$$

$$\rho_i\frac{\partial e_i}{\partial t}+\rho_i u_i\frac{\partial e_i}{\partial x}+p_i\frac{\partial u_i}{\partial x}=\sum_{j(i\neq j)}q_{ij}(T_j-T_i)+\sum_{j(i\neq j)}\beta_{ij}K_{ij}(u_j-u_i)^2+\frac{4}{3}\mu_i\Big(\frac{\partial u_i}{\partial x}\Big)^2+\frac{\partial}{\partial x}\Big(\lambda_i\frac{\partial T_i}{\partial x}\Big), \tag{1.1}$$

$$p_i=R_i\rho_iT_i,\qquad e_i=c_{iv}T_i,$$

where ρ_i, u_i, and T_i are the mass density, velocity, and temperature of the ith component, $R_i=k/m_i$ (k is the Boltzmann constant and m_i is the molecular mass of the ith component), and $n_i=\rho_i/m_i$ is the numerical density of the ith component. The coefficients K_{ij} and q_{ij}, which characterize the momentum and energy exchange between the components of the mixture, and the partial coefficients of viscosity μ_i and thermal conductivity λ_i for a binary mixture, have the following form [1]:

[1]Novosibirsk State University of Architecture and Civil Engineering, Novosibirsk 630008. [2]Institute of Theoretical and Applied Mechanics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 4, pp. 47–57, July–August, 2002. Original article submitted March 25, 2002.

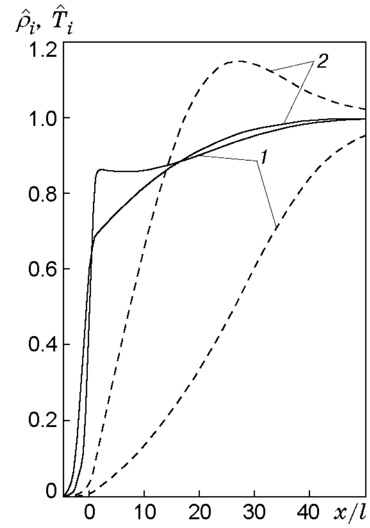

Fig. 1

$$K_{12} = \frac{16}{3}\,\frac{\rho_1\rho_2}{m_1+m_2}\,\Omega_{12}^{(1,1)}, \quad q_{12} = \frac{3k}{m_1+m_2}\,K_{12}, \quad \mu_1 = \frac{5}{8}\,kT_1\Big(\Omega_1^{(2,2)} + 2\,\frac{n_2}{n_1}\,\Omega_{12}^{(2,2)}\Big)^{-1},$$

$$\mu_2 = \frac{5}{8}\,kT_2\Big(\Omega_2^{(2,2)} + \frac{20}{3}\,\frac{\rho_1}{\rho_2}\,\Omega_{12}^{(1,1)}\Big)^{-1}, \qquad \lambda_1 = \frac{75}{32}\,\frac{k^2T_1}{m_1}\Big(\Omega_1^{(2,2)} + 5\,\frac{n_2}{n_1}\,\Omega_{12}^{(1,1)}\Big)^{-1}, \tag{1.2}$$

$$\lambda_2 = \frac{75}{32}\,\frac{k^2T_2}{m_2}\Big(\Omega_2^{(2,2)} + 15\,\frac{\rho_1}{\rho_2}\,\Omega_{12}^{(1,1)}\Big)^{-1}.$$

The subscripts 1 and 2 refer to the light and heavy components, respectively; $\Omega_{ij}^{(k,l)}$ are the collision integrals.

We have to find a steady-state solution of system (1.1) that satisfies the boundary conditions

$$(\rho_i, u_i, T_i) \to (\rho_i^0, u^0, T^0) \quad \text{as} \quad x \to -\infty, \quad (\rho_i, u_i, T_i) \to (\rho_i^1, u^1, T^1) \quad \text{as} \quad x \to +\infty. \tag{1.3}$$

The superscript 0 corresponds to the free-stream parameters, and the quantities marked by the superscript 1 are related to the free-stream parameters by the Hugoniot relations for an equilibrium mixture.

Problem (1.1), (1.3) was solved numerically by the pseudotransient method with the use of an implicit scheme of splitting in accordance with physical processes for a binary mixture of gases in [1] and, for a ternary mixture of gases, in [2]. Satisfactory agreement was reached with experimental data and calculations by kinetic equations and the Direct Simulation Monte Carlo (DSMC) method for Mach numbers M $= 1$–4 and wide ranges of molar concentrations of the components and ratios of molecular masses.

Some features of the shock-wave structure were noted in [1, 2]. In particular, for a low concentration of the heavy gas in the binary mixture, a plateau can appear on the density profile of the light component. This effect was also observed in experiments [3] and kinetic calculations [4]. Figure 1 shows an example of such a calculation with the use of system (1.1) in a 99% He + 1% Xe mixture for a Mach number ahead of the shock wave $\mathrm{M}_0 = 3$. The dimensionless densities and temperatures of the components are calculated by the formulas $\hat{\rho}_i = (\rho_i - \rho_i^0)/(\rho_i^1 - \rho_i^0)$ and $\hat{T}_i = (T_i - T^0)/(T^1 - T^0)$. Figure 1 shows the density profiles $\hat{\rho}_i$ (curves 1) and temperature profiles $\hat{T}_i$ (curves 2) of the components; the solid and dashed curves refer to the light and heavy components, respectively (l is the mean free path of a molecule determined in [5]). First, the density of the light gas increases, then there is a section of an almost constant density, and after that a more dramatic increase in density is observed. The calculations were performed using the model of hard spheres. As is shown in [1], similar results were obtained for different power potentials of intermolecular interaction, where the power exponents were chosen on the basis of experimental data.

Another special feature of the shock-wave structure in a binary mixture is the nonmonotonic behavior of temperature of the heavy gas in the case of a low molar concentration of the latter. The temperature of the heavy component increases to a certain value higher than the equilibrium temperature behind the shock wave and then decreases back, tending to the equilibrium temperature.

We consider a binary mixture. We assume that $K = K_{12}$, $\beta_i = \beta_{ij}$ $(j \neq i)$, $q = q_{ij}$, and μ_i, λ_i are positive constants. For a mixture of monatomic gases, we have $\gamma_1 = \gamma_2 = \gamma = R_i/c_{iv}$. We assume that all the sought functions of system (1.1) depend on the coordinate $\xi = x - Dt$ (D is the shock-wave velocity). We introduce dimensionless variables by the formulas

$$\bar{V}_i = (c_1 + c_2)V_i/c_3, \quad \bar{T}_i = (R_1c_1 + R_2c_2)(c_1 + c_2)T_i/c_3^2, \quad \bar{\rho}_i = c_3\rho_i/(c_1 + c_2)^2,$$

$$\bar{\mu}_i = \mu_i/\mu_*, \quad \bar{\lambda}_1 = \lambda_1/\lambda_*, \quad \bar{\xi} = \xi(c_1 + c_2)/\mu_*, \quad \mu_* = (c_1 + c_2)^2/K,$$

$$\lambda_* = (R_1c_1 + R_2c_2)\mu_*/(c_1 + c_2), \quad \bar{q} = \mu_* q/((c_1 + c_2)(R_1c_1 + R_2c_2)),$$

where c_i are constants of integration and $V_i = u_i - D$.

Using the dimensionless variables (in what follows, the bar is omitted), system (1.1) is transformed to

$$\rho_i V_i = \alpha_i^0, \quad \alpha_1^0 V_1 + \alpha_2^0 V_2 + x_1^0 \frac{T_1}{V_1} + x_2^0 \frac{T_2}{V_2} = 1 + \frac{4}{3}\mu_1 \frac{dV_1}{d\xi} + \frac{4}{3}\mu_2 \frac{dV_2}{d\xi},$$

$$\frac{\gamma}{\gamma - 1}(x_1^0 T_1 + x_2^0 T_2) + \frac{\alpha_1^0 V_1^2 + \alpha_2^0 V_2^2}{2} = A + \frac{4}{3}\mu_1 V_1 \frac{dV_1}{d\xi} + \frac{4}{3}\mu_2 V_2 \frac{dV_2}{d\xi} + \lambda_1 \frac{dT_1}{d\xi} + \lambda_2 \frac{dT_2}{d\xi},$$

$$\alpha_2^0 \frac{dV_2}{d\xi} + x_2^0 \frac{d(T_2/V_2)}{d\xi} = V_1 - V_2 + \frac{4}{3}\mu_2 \frac{dV_2}{d\xi}, \tag{1.4}$$

$$\frac{x_2^0}{\gamma - 1}\frac{dT_2}{d\xi} + x_2^0 \frac{T_2}{V_2}\frac{dV_2}{d\xi} = \beta_2(V_1 - V_2)^2 + q(T_1 - T_2) + \frac{4}{3}\mu_2\Big(\frac{dV_2}{d\xi}\Big)^2 + \lambda_2 \frac{d^2T_2}{d\xi^2}.$$

Here $A = (\gamma + 1)V^0V^1/(2(\gamma - 1))$, $V^0 = \gamma \mathrm{M}_0^2/(\gamma \mathrm{M}_0^2 + 1)$, $V^1 = 2\gamma/(\gamma + 1) - V^0$, and α_i^0 is the mass concentration of the ith component of the mixture ahead of the shock wave.

We have to find a solution of system (1.4) that satisfies the boundary conditions

$$V_i \to V^0, \quad T_i \to T^0 \quad \text{as} \quad \xi \to -\infty, \qquad V_i \to V^1, \quad T_i \to T^1 \quad \text{as} \quad \xi \to +\infty, \tag{1.5}$$

where $T^i = V^i(1 - V^i)$.

2. Asymptotic Study in the Presence of a Plateau. We consider the case of a low molar concentration of the heavy gas and a finite mass concentration ($x_2^0 \ll 1$). In this approximation, the mass of heavy gas molecules is much greater than the mass of light gas molecules. In the Euler approximation ($\mu_i = \lambda_i = 0$), from (1.4), we obtain the following system of equations:

$$L(V_1, V_2) = \frac{\gamma + 1}{2\gamma}\alpha_1^0 V_1^2 + \alpha_2^0 V_1 V_2 - \frac{\gamma - 1}{2\gamma}\alpha_2^0 V_2^2 - V_1 + \frac{\gamma - 1}{\gamma}A = 0, \quad \frac{dV_2}{d\tau} = V_1 - V_2,$$

$$T_1 = \frac{\gamma - 1}{\gamma}\Big(A - \frac{\alpha_1^0 V_1^2}{2} - \frac{\alpha_2^0 V_2^2}{2}\Big), \quad \frac{dT_2}{d\tau} + \frac{(\gamma - 1)T_2}{V_2}\frac{dV_2}{d\tau} = F_2(V_1 - V_2)^2 + F_3(T_1 - T_2). \tag{2.1}$$

Here $\tau = \xi/\alpha_2^0$, $F_2 = \beta_2\alpha_2^0(\gamma - 1)/x_2^0 = ((\gamma - 1)m_2/m_1)/((1 + m_2/m_1)(x_1^0 + x_2^0 m_2/m_1))$, and $F_3 = q(\gamma - 1)\alpha_2^0/(x_2^0 K) = (3(\gamma - 1)m_2/m_1)/(1 + m_2/m_1)$.

In deriving system (2.1), we retain quantities of the order of $x_2^0\, df/d\xi$; in this case, we have $\alpha_2^0\, dV_2/d\xi \gg x_2^0\, d(T_2/V_2)/d\xi$.

The results of an analysis of the first equation in (2.1) in the plane $(\hat{V}_1, \hat{V}_2)$ [$\hat{V}_i = (V_i - V^1)/(V^0 - V^1)$] are plotted in Fig. 2. The data are obtained for the conditions $\mathrm{M}_0^2 < 1/\alpha_1^0$ (curve 1; the shock-wave velocity is smaller than the velocity of sound in the light gas), $\mathrm{M}_*^2 = 2\gamma/((\gamma - 1)\alpha_1^0) > \mathrm{M}_0^2 > 1/\alpha_1^0$ (curve 2), and $\mathrm{M}_0 > \mathrm{M}_*$ (curve 3). Thus, if the shock-wave velocity does not exceed the velocity of sound in the light gas, a continuous solution is obtained; otherwise, the solution has a discontinuity at which the heavy gas velocity is continuous. If the shock-wave velocity is smaller than a certain value, both velocities decrease monotonically. If the shock-wave velocity is higher than this value, however, the velocity of the light component increases immediately behind the discontinuity, reaches a maximum, and then decreases, tending to the equilibrium velocity behind the shock wave.

Similar results were obtained in [6], where relaxation-type equations for a binary mixture in the one-temperature approximation were considered. In this case, the pressure of the mixture is a rigorously increasing function

$$\frac{dP}{d\tau} = \frac{\alpha_2^0[1 - V_2 + (\alpha_1^0/\gamma)(V_2 - V_1)]}{\partial L/\partial V_1}(V_1 - V_2) > 0,$$

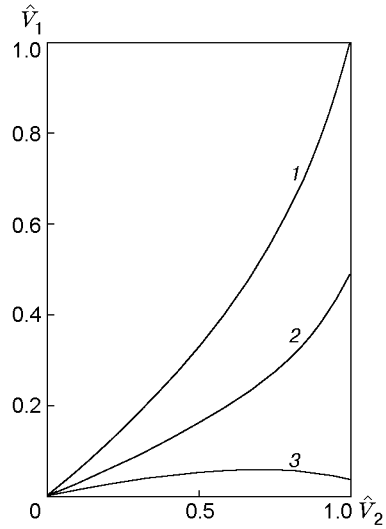

Fig. 2

since $V_2 < 1$ and $V_2 > V_1$, and the derivative is $\partial L/\partial V_1 < 0$ on the curve $L(V_1, V_2) = 0$ in the domain considered (Fig. 2).

Thus, the light gas is accelerated in the region with an elevated pressure. Equation (2.1) yields

$$\alpha_1^0 \frac{dV_1}{d\xi} = -\frac{dP}{d\xi} + \alpha_2^0(V_2 - V_1),$$

i.e., the force of intercomponent exchange immediately behind the discontinuity is greater than the pressure gradient, which is the reason for the light gas acceleration.

The anomalous behavior of the light component velocity may be caused by the neglect of viscosity. Let $\mu_1 \neq 0$, then, the problem of velocity determination reduces to a system of two ordinary differential equations

$$F_1 V_1 \frac{dV_1}{d\tau} = \gamma L(V_1, V_2), \qquad \frac{dV_2}{d\tau} = V_1 - V_2,$$

where $F_1 = (4/3)(\mu_1 K/\alpha_2^0)$.

We have to find a solution connecting the equilibrium positions (V^0, V^0) and (V^1, V^1). We determine the type of these singularities. Linearization in the vicinity of (V^0, V^0) yields the characteristic equation

$$\lambda^2 + \lambda\Big(1 - \frac{(\gamma + \alpha_1^0)V^0 - \gamma}{F_1 V^0}\Big) - \frac{(\gamma + 1)V^0 - \gamma}{F_1 V^0} = 0.$$

In the shock wave, $\mathrm{M}_0 > 1$ and, hence, $V^0 > \gamma/(\gamma + 1)$. Therefore, the point (V^0, V^0) is a saddle. The output direction corresponding to positive λ has the form $k^{(0)} = (B + \sqrt{B^2 + 4\alpha_2^0/F_1}\,)/2$, where $B = 1 + ((\gamma + \alpha_1^0) \times V^0 - \gamma)/(F_1 V^0)$.

Similarly, for the point (V^1, V^1), we obtain

$$\lambda^2 + \lambda\Big(1 - \frac{(\gamma + \alpha_1^0)V^1 - \gamma}{F_1 V^1}\Big) - \frac{(\gamma + 1)V^1 - \gamma}{F_1 V^1} = 0.$$

Taking into account that $V^1 < \gamma/(\gamma + 1)$ and $\alpha_1^0 < 1$, we can easily show that $\lambda_1 < 0$ and $\lambda_2 < 0$, i.e., this singular point (V^1, V^1) is a node. The input direction of integral curves corresponding to the minimum eigenvalue is determined by the relation $k^{(1)} = (C + \sqrt{C^2 + 4\alpha_2^0/F_1}\,)/2$, where $C = 1 + (\gamma - (\gamma + \alpha_1^0)V^1)/(F_1 V^1)$.

We consider three cases depending on the behavior of the isocline $L(V_1, V_2) = 0$.

Case 1: $\mathrm{M}_0^2 < 1/\alpha_1^0$. In this case, the integral curve emanating from the point (V^0, V^0) enters the region bounded by isoclines of vertical and horizontal inclinations $V_1 - V_2 = 0$ and $L(V_1, V_2) = 0$, respectively, and acquires an equilibrium state (V^1, V^1). This behavior of integral curves follows from the inequalities

$$1 < k^{(0)} < k_L^{(0)}, \qquad 1 > k^{(1)} > k_L^{(1)}, \qquad \frac{dV_2}{d\tau} < 0.$$

Here $k_L^{(i)} = -(\partial L/\partial V_2)/(\partial L/\partial V_1)$ for $V_1 = V_2 = V^i$.

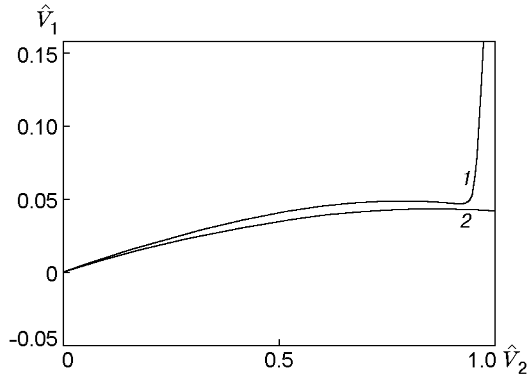

Fig. 3

Case 2: $M_0^2 > 1/\alpha_1^0$. The integral curve is located in the domain bounded by the curves $V_1 - V_2 = 0$, $L(V_1, V_2) = 0$, and $V_2 = V^0$ for $M_0 < M_*$ and by the curves $V_1 - V_2 = 0$, $V_1 = V^1$, and $V_2 = V^0$ for $M_0 > M_*$, which follows from the inequalities and relations

$$1 < k^{(0)} < \infty, \qquad 1 < k^{(1)} < k_L^{(1)}, \qquad \frac{dV_2}{d\tau} < 0,$$

$$\frac{dV_1}{dV_2} = \frac{\gamma+1}{2F_1V_1}\,(V_1 - V_1^+) \quad \text{for} \quad V_2 = V^0, \quad \frac{dV_1}{dV_2} = -\frac{\alpha_2^0(\gamma+1)}{2F_1V^1}\Big(V_2 - \frac{\gamma-1}{\gamma+1}\,V^1\Big) \quad \text{for} \quad V_1 = V^1.$$

Here $V^+ = V^0 + 2(\gamma - (\gamma+\alpha_1^0)V^0)/((\gamma+1)\alpha_1^0)$ is the light gas velocity behind the shock in the Euler approximation.

Thus, for all F_1, the only integral curve emanating from the point (V^0, V^0) cannot leave the corresponding domains and inevitably acquires the steady state (V^1, V^1), i.e., there exists a solution of the initial problem, and this solution is unique.

In the first two cases, the velocities of both components decrease monotonically.

Case 3: $M_0 > M_*$. In this case, the behavior of the light gas velocity depends on F_1. If $F_1 \geqslant 1$, the velocities are monotonic, i.e., allowance for viscosity may eliminate the nonmonotonic character of the light gas velocity. If $F_1 \ll 1$, the integral curve emanating from the point (V^0, V^0) is first located in a narrow band $[V_2^0 - \varepsilon, V_2^0]$ and then reaches the isocline of zero inclinations $L(V_1, V_2) = 0$. As a result, the derivative of velocity of the light component vanishes. Then, the light gas velocity increases until the integral curve intersects the isocline again; after that, the velocity decreases, tending to the equilibrium velocity behind the shock wave (curve 1 in Fig. 3).

For the model of elastic hard spheres for $x_2 \ll 1$ and $m_1/m_2 \ll 1$, Eq. (1.2) yields $F_1 \approx m_1/(m_2 M_0^2)$, i.e., $F \ll 1$. For example, for a He–Xe mixture, we have $F_1 \approx 0.004$ for $M_0 = 3$. Hence, under real conditions, the light gas velocity in the shock wave has two extreme points for $M_0 > M_*$. In calculations of the shock-wave structure, this behavior corresponds to the emergence of a plateau. The reason is the small difference in velocities at extreme points. The greatest difference is reached in the Euler approximation. The estimates show that $(V_* - V_1^+)/(V^0 - V^1) \approx 0.015$ (V_* is the light gas velocity at the point of a local maximum) with increasing Mach number up to 6 and ratio of molecular masses up to 100. Curve 1 in Fig. 3 shows the dependence $\hat{V}_1(\hat{V}_2)$ in the plateau region, which was calculated using the full model with parameters corresponding to Fig. 1; in this case, $M_0 > M_*$. In calculations by the full model, the velocity profile has also two extreme points. Curve 2 in Fig. 3 shows the calculation results obtained by formulas (2.1). In the relaxation region, Eqs. (2.1) give an adequate description of the nonmonotonic profile of the light gas velocity.

3. Nonmonotonicity of the Temperature Profile of the Heavy Component. The nonmonotonicity of the temperature profile of the heavy gas was previously obtained in [1, 7]. Bird [7] gives the DSMC results for the dependence of the temperature peak of the heavy component on the molar concentration, ratio of molecular masses, and Mach number.

The effect of nonmonotonicity can be explained using Eqs. (2.1). Figure 4 shows a comparison of the temperature distributions $\hat{T}_1(\hat{V}_2)$ (solid curves) and $\hat{T}_2(\hat{V}_2)$ (dashed curves) for $M_0 = 3$ and $x_2^0 = 0.01$ in the He–Xe mixture [curves 1 and 2 show the calculations by the full model and Eqs. (2.1), respectively]. In the relaxation region, these dependences are in good agreement. It follows from the last equation in (2.1) that the temperature of the heavy gas changes as a result of deceleration processes in the shock wave, energy dissipation (work of the

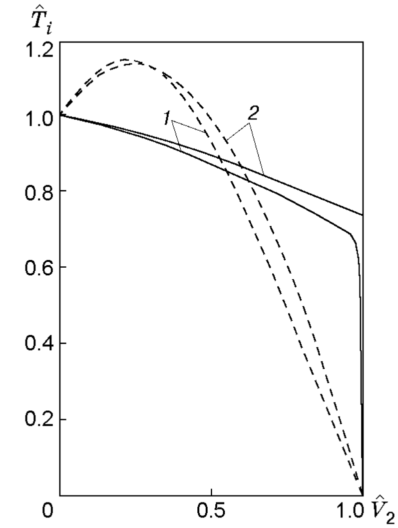

Fig. 4

force of intercomponent interaction), and energy exchange between the components because of the temperature difference. The influence of the first two factors leads to an increase in temperature of the heavy gas in the shock wave; in addition, $T_1 > T_2$ at the initial section. As a result, the temperature of the heavy gas becomes higher than the temperature of the light gas and may exceed the equilibrium temperature behind the shock wave; after that, it tends to the equilibrium temperature behind the shock wave due to heat exchange with the light gas.

Formulas (2.1) can be simplified in the case of low molar and mass concentrations of the heavy gas.

Let $x_2^0 \ll \alpha_2^0 \ll 1$. Assuming that $\lambda_i = \mu_2 = 0$, we obtain from (1.4)

$$F_1 V_1 \frac{dV_1}{d\tau} = \frac{\gamma+1}{2}(V_1 - V^0)(V_1 - V^1), \qquad T_1 = (\gamma - 1)\Big(\frac{V_1^2}{2} - V_1 + A\Big),$$
$$\frac{dV_2}{d\tau} = V_1 - V_2, \qquad \frac{dT_2}{d\tau} = -\frac{(\gamma-1)T_2}{V_2}\frac{dV_2}{d\tau} + F_2(V_1 - V_2)^2 + F_3(T_1 - T_2) \tag{3.1}$$

(F_i are defined in Sec. 2).

We have to find a solution of (3.1) satisfying the boundary conditions (1.5). In the general case, the solution of this problem is

$$\frac{V^0}{V^0 - V^1}\ln(V^0 - V_1) - \frac{V^1}{V^0 - V^1}\ln(V_1 - V^1) = \frac{\gamma+1}{2F_1}\tau + \ln\frac{V^0 - V^1}{2},$$
$$T_1 = (\gamma-1)\Big(A + \frac{V_1^2}{2} - V_1\Big), \qquad V_2 = \exp(-\tau)\int\limits_{-\infty}^{\tau} V_1(y)\exp(y)\,dy, \tag{3.2}$$

$$T_2 = V_2^{-(\gamma-1)}(\tau)\exp(-F_3\tau)\Bigg(F_3\int\limits_{-\infty}^{\tau} V_2^{\gamma-1}(y)T_1(y)\exp(F_3 y)\,dy + F_2\int\limits_{-\infty}^{\tau}(V_1(y) - V_2(y))^2 V_2^{\gamma-1}(y)\exp(F_3 y)\,dy\Bigg).$$

Since the temperature peak is in the relaxation region, we can pass to the limit at $F_1 \to 0$ in (3.2). As a result, we obtain

$$V_i = V^0, \quad T_i = T^0 \quad \text{for} \quad \tau < 0,$$
$$V_1 = V^1, \quad T_1 = T^1, \quad V_2 = \varphi(\tau), \quad T_2 = \phi(\tau) \quad \text{for} \quad \tau > 0,$$
$$\varphi(\tau) = V^1 + (V^0 - V^1)\exp(-\tau), \tag{3.3}$$

$$\phi(\tau) = \varphi^{-(\gamma-1)}\exp(-F_3\tau)\Bigg((V^0)^{\gamma-1}T^0 + F_3\int\limits_0^{\tau}\varphi^{\gamma-1}T^1\exp(F_3 y)\,dy + F_2\int\limits_0^{\tau}(V^1 - \varphi)^2\varphi^{\gamma-1}\exp(F_3 y)\,dy\Bigg).$$

TABLE 1

M_0	m_2/m_1	α_2^0	$(T^* - T^1)/(T^1 - T^0)$, %		
			Full model	(2.1)	(3.3)
4	10	0.0917	29	24	26
	50	0.3356	15	10	21
	100	0.5025	8	6	18
3	10	0.0917	27	23	26
	50	0.3356	13	10	23
	100	0.5025	6	4	20
2	10	0.0917	20	20	25
	50	0.3356	8	6	21
	100	0.5025	3	2	19

The calculation results for the temperature peak T^* of the heavy gas for $x_2^0 = 0.01$ are listed in Table 1. One can see that Eqs. (3.3) yield rather exact values of the temperature peak only for a low mass concentration of the heavy gas.

Relations (3.3) become much simpler if $V^0 - V^1 = \varepsilon \ll 1$ (weak shock waves). Obviously, in the approximation considered, we have $F_1 \ll \varepsilon$. We introduce new dimensionless velocities v_i and temperatures t_i:

$$V_i = \gamma/(\gamma+1) + (\varepsilon/2)v_i, \qquad T_i = \gamma/(\gamma+1)^2 + (\gamma-1)/(2(\gamma+1))\varepsilon t_i - \varepsilon^2/4. \tag{3.4}$$

From (3.3), in the zeroth approximation in terms of ε, we obtain

$$v_i = 1, \qquad t_i = -1 \quad \text{for} \quad \tau < 0,$$

$$v_1 = -1, \quad t_1 = 1, \quad v_2 = -1 + 2\exp(-\tau), \quad t_2 = 1 + 2/(F_3 - 1)w(1 - F_3 w^{F_3 - 1}) \quad \text{for} \quad \tau > 0,$$

$$w = (1 + v_2)/2.$$

If the ratio of molecular masses tends to zero and $\gamma = 5/3$, then from (2.1), we obtain $F_3 \approx 2$, i.e., the dependence of the temperature of the heavy gas on its velocity is quadratic. The temperature peak is 12.5% of the overall pressure difference in the shock wave.

4. Shock-Wave Structure in a Ternary Mixture of Gases. We consider a flow of a ternary mixture of gases, consisting of a light gas and two heavy gases. The subscripts 1, 2, and 3 refer to the light, less heavy, and heavy gases, respectively. The system of equations of three-velocity, three-temperature gas dynamics in the one dimensional case has the form (1.1), where the coefficients K_{ij} and q_{ij} characterizing the momentum and energy exchange between the components of the mixture and the partial coefficients of viscosity and thermal conductivity are set by expressions given in [1]. Since these expressions are rather cumbersome, we confine ourselves to the case of low concentrations of the heavy gases $n_2 \approx n_3 \ll n_1$ and $m_1/m_i \ll 1$ $(i = 2, 3)$, i.e., $\rho_1 \approx \rho_2 \approx \rho_3$. Then, the expressions for the kinetic coefficients take the form

$$K_{1i} = \frac{16}{3}\rho_1 n_i \Omega_{1i}^{(1,1)}, \quad q_{1i} = \frac{3k}{m_i} K_{1i} \quad (i = 2, 3), \quad K_{23} = \frac{16\rho_2\rho_3}{3(m_2 + m_3)} \Omega_{23}^{(1,1)},$$
$$\mu_1 = \frac{5kT_1}{8\Omega_1^{(2,2)}}, \quad \mu_i = \frac{3k\rho_i T_i}{32\rho_1 \Omega_{1i}^{(1,1)}}, \quad \lambda_1 = \frac{15k}{4m_1}\mu_1, \quad \lambda_i = \frac{5k}{3m_i}\mu_i. \tag{4.1}$$

Calculations of the shock-wave structure in the ternary mixture of gases [2] show that the velocities and temperatures of the components of the mixture in the shock wave are significantly different. As in the binary mixture, the temperature profiles of the heavy components are nonmonotonic (there may be temperature peaks in both heavy gases or only in one of them). Mixtures with the ratio of molecular masses of the heavy components of the order of unity were considered.

In the present work, we calculated the shock-wave structure in a ternary mixture of gases with $m_3/m_2 \gg 1$. The calculation results for $M_0 = 3$, $x_2^0 = 0.01$, $x_3^0 = 0.0025$, $m_2/m_1 = 10$, and $m_3/m_2 = 10$ are plotted in Fig. 5 (curves 1, 2, and 3 refer to the light, less heavy, and heavy gases, respectively). In addition to the above-described effects, some new effects were observed is such mixtures. First, the plateau may be in the light gas only

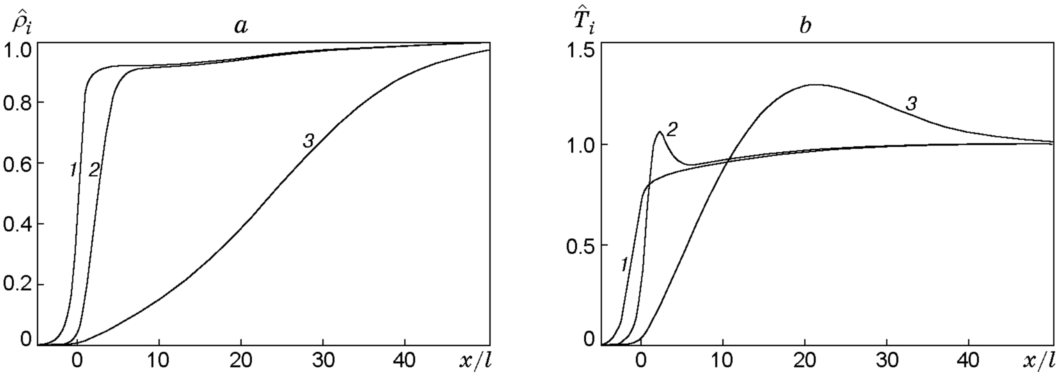

Fig. 5

or simultaneously in the light and less heavy gases (Fig. 5a). This occurs if there are two clearly expressed zones in the relaxation region: the zone of equalization of velocities and temperatures of the light and less heavy gases and a more extended zone where the mixture acquires the equilibrium state. Second, the temperature profile of the second component (less heavy gas) may have two local extreme points. The temperature of the second component first increases and becomes greater than the temperature of the light gas, then it decreases to a certain value and increases again, tending to the equilibrium temperature behind the shock wave but remaining higher than the temperature of the light gas (Fig. 5b). The temperature of the second component at the point of a local maximum may be either higher or lower than the equilibrium temperature behind the shock wave. This effect is caused by the presence of the third component. The temperature of the second component decreases because of the intense heat exchange with the heavy gas whose temperature is significantly lower.

To construct asymptotic solutions, we pass to dimensionless velocities and temperatures, using formulas similar to those in Sec. 1, where $K = K_{13}$. We obtain

$$\rho_i V_i = \alpha_i^0, \quad \sum_{i=1}^{3} \Big(\alpha_i^0 V_i + x_i^0 \,\frac{T_i}{V_i} - \frac{4}{3}\,\mu_i \,\frac{dV_i}{d\xi}\Big) = 1,$$

$$\sum_{i=1}^{3} \Big(\frac{\gamma}{\gamma-1}\,x_i^0 T_i + \frac{\alpha_i^0 V_i^2}{2} - \frac{4}{3}\,\mu_i V_i \,\frac{dV_1}{d\xi} - \lambda_i \,\frac{dT_i}{d\xi}\Big) = \tilde{A},$$

$$\alpha_i^0 \,\frac{dV_i}{d\xi} + x_i^0 \,\frac{d(T_i/V_i)}{d\xi} = K_{1i}(V_1 - V_i) + K_{ij}(V_j - V_i) + \mu_i \,\frac{d^2 V_i}{d\xi^2}, \tag{4.2}$$

$$\frac{x_i^0}{\gamma-1}\,\frac{dT_i}{d\xi} + x_i^0 \,\frac{T_i}{V_i}\,\frac{dV_i}{d\xi} = K_{1i}\beta_{i1}(V_1 - V_i)^2 + K_{ij}\beta_{ij}(V_j - V_i)^2$$

$$+ q_{1i}(T_1 - T_i) + q_{ij}(T_j - T_i) + \frac{4}{3}\,\mu_i \Big(\frac{dV_i}{d\xi}\Big)^2 + \lambda_i \,\frac{d^2 T_i}{d\xi^2} \quad (i,j = 2,3,\ i \neq j),$$

where $\tilde{A} = c_5(c_1 + c_2 + c_3)/c_4^2$ (c_i are constants of integration).

We have to find a solution of system (4.2) satisfying conditions (1.5).

We consider the approximation $x_i^0 \ll \alpha_i^0 \ll 1$ ($i = 2, 3$), $\mu_2 = \mu_3 = \lambda_i = 0$. From (4.1), there follows

$$K_{23}/K_{12} \approx x_3^0 \sqrt{m_2/m_1} \ll 1, \quad K_{23}/K_{13} \approx x_2^0 \sqrt{m_2/m_1} \ll 1, \quad q_{23}/q_{12} \ll 1, \quad q_{23}/q_{13} \ll 1.$$

From (4.2), in the zeroth approximation, we obtain

$$V_1 + \frac{T_1}{V_1} = 1 + \mu_1 \frac{dV_1}{d\xi}, \quad T_1 = (\gamma - 1)\Big(\frac{V_1^2}{2} - V_1 + \tilde{A}\Big), \quad \alpha_i^0 \frac{dV_i}{d\xi} = K_{1i}(V_1 - V_i),$$

$$x_i^0 \frac{dT_i}{d\xi} = -\frac{T_i}{V_i}\frac{dV_i}{d\xi} + K_{1i}\beta_{1i}(V_1 - V_i)^2 + q_{1i}(T_1 - T_i), \quad i, j = 2, 3, \ i \neq j. \tag{4.3}$$

Thus, for determining the parameters of the light gas, we have equations similar to those for the one-component gas. To find the parameters of the heavy gases, we obtain relaxation-type equations. By integrating system (4.3) in an explicit form, we obtain formulas similar to (3.2). Passing to the limit as $\mu_1 \to 0$ and confining ourselves to the case of weak shock waves, we introduce new dimensionless velocities and temperatures, using Eqs. (3.4). In the zeroth approximation, we obtain

$$v_i = 1, \qquad t_i = -1 \qquad \text{for} \quad \tau < 0,$$

$$v_1 = -1, \quad t_1 = 1, \quad v_2 = -1 + 2\exp(-F\tau), \quad v_3 = -1 + 2\exp(-\tau),$$

$$t_2 = 1 - 2w^{3(\gamma-1)F} + 2(w^F - w^{3(\gamma-1)F})/(3\gamma - 4), \quad t_3 = 1 - 2w^{3(\gamma-1)} + 2(w - w^{3(\gamma-1)})/(3\gamma - 4),$$

$$F = (m_3/m_2)(\sigma_{12}/\sigma_{13})^2, \quad w = (1 + v_3)/2, \quad \sigma_{1i} = (\sigma_1 + \sigma_i)/2 \quad \text{for} \quad \tau > 0,$$

where σ_i is the diameter of a molecule of the ith gas.

The behavior of velocities and temperatures of the heavy gases in the approximation considered depends on the ratio of masses and diameters of their molecules. For $F < 1$, the temperature peak of the less heavy gas is located on the right of the temperature peak of the heavy gas. With increasing F, the temperature peak of the less heavy gas is shifted; for $F \gg 1$, it is located in the beginning of the relaxation region, and the temperature peak of the heavy gas is located at the end of the relaxation region, which is in qualitative agreement with the calculation results by the full model.

Thus, asymptotic solutions of the problem of the shock-wave structure have been obtained for the model of multivelocity, multitemperature gas dynamics of gas mixtures. The emergence of a plateau on the density profile of the light component and nonmonotonicity of temperature profiles of the heavy gases have been explained on the basis of these solutions.

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